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## Brevia

## SHORT NOTES

# The dilation direction of intrusive sheets

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**Abstract**—Where a dilation dike or sill cuts two non-parallel planar features, the direction of dilation can be determined by: (1) orthographic projection; (2) equal-area projection (Schmidt net); or (3) analytical geometry.

OFFSET criteria are commonly used to determine if a dike (or sill) has formed during dilation or by replacement of country rocks (Goodspeed 1940). If dilation can be demonstrated, a question arises concerning the direction of movement of one wall relative to the other during dilation. This problem is most readily solved if the dike cuts two non-parallel planes, for example a bedding plane and an earlier dike. The orientation of a dilation line (Kretz 1968, p. 43), that is a straight line joining two initially adjacent points on opposite walls of the dike, can then be determined.

Recently Bussell (1989) solved the dilation problem by orthographic projection (method 1; Fig. 1a) and noted that under favorable circumstances the method is simple and rapid.

The problem may also be solved by equal-area projection (method 2), that is by use of the Schmidt net (Kretz 1968, p. 43). Referring to Fig. 1(a), an infinite number of parallel dilation lines must lie within a plane ($a/AC$) defined by the line of intersection of planes $A$ and $C$ (line $AC$) and line $a$ which lies in the plane of observation and is referred to as an offset line. Similarly, an infinite number of dilation lines lie within plane ($b/BC$) defined by the line of intersection of planes $B$ and $C$ (line $BC$) and offset line $b$ (Fig. 1b). The line of intersection of these two planes locates a measurable dilation line. The bearing and plunge (106°/32°) of this line are then easily measured on the Schmidt net. In this solution, offset lines $a$ and $b$ need not be horizontal as they are in Fig. 1(a); that is the plane of observation is not necessarily horizontal.

A supplementary calculation gives the length of the

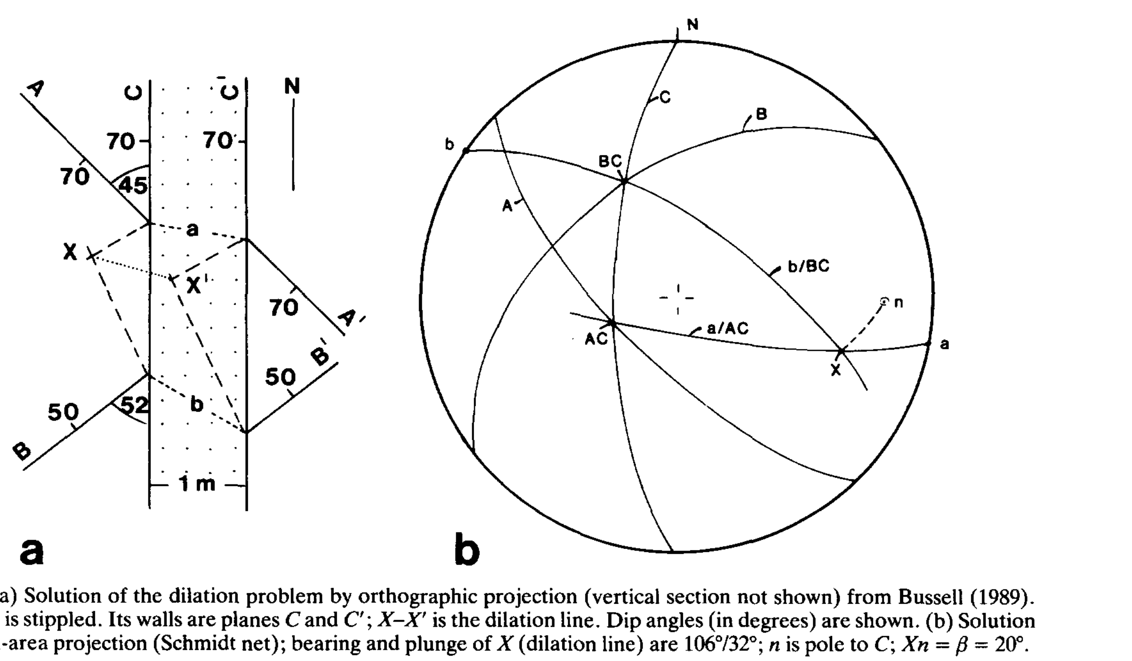


Fig. 1. (a) Solution of the dilation problem by orthographic projection (vertical section not shown) from Bussell (1989). The dike is stippled. Its walls are planes $C$ and $C'$; $X$–$X'$ is the dilation line. Dip angles (in degrees) are shown. (b) Solution by equal-area projection (Schmidt net); bearing and plunge of $X$ (dilation line) are 106°/32°; $n$ is pole to $C$; $Xn = \beta = 20°$.

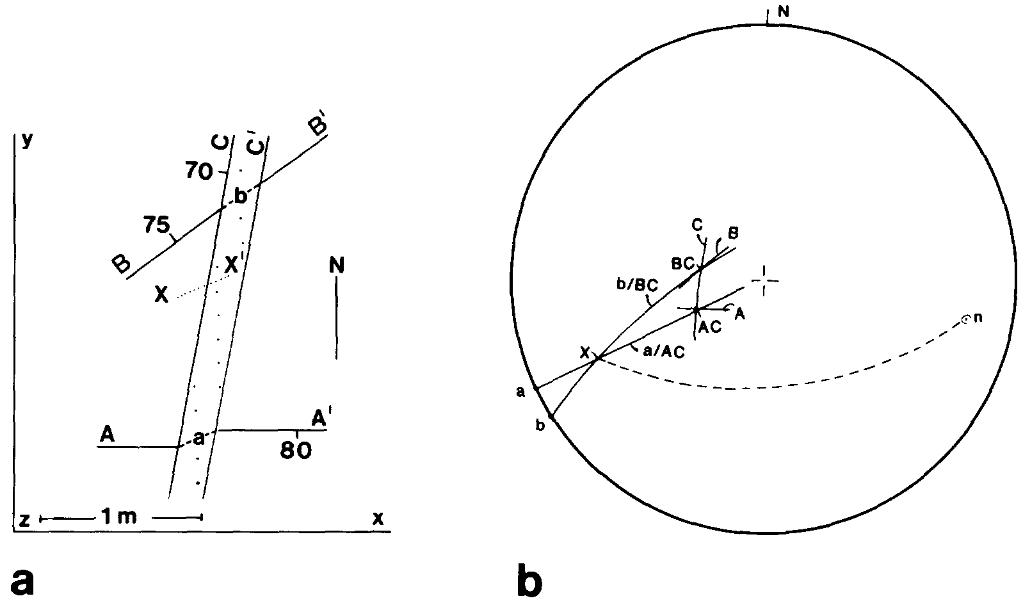


Fig. 2. (a) Pegmatite dike C cutting pegmatite dikes A and B (from Kretz 1968). (b) Bearing and plunge of $X$ (dilation line) are 245°/28° for dike C, obtained by equal-area projection (Schmidt net); $n$ is pole to $C$; $Xn = \beta = 120°$ or 60°.

measurable dilation line (denoted $s$) from the apparent dike width ($w'$), dike dip ($\alpha$) and the angle ($\beta$) between the dilation line and the normal ($n$) to the dike plane, as obtained from the Schmidt net:

$$s = (w' \sin \alpha)/(\cos \beta).$$

With regard to Fig. 1, $\alpha = 70°$, $\beta = 20°$ (Fig. 1b) and if $w' = 1.0$ m, it follows that $s = 1.0$ m, in agreement with Bussell's solution by method 1.

Method 2 applied to a pegmatite dike near Yellowknife (Fig. 2) gives 245°/28° and 0.38 m as the bearing/plunge and length of the dilation line. For some dikes (Kretz 1968, fig. 11), lines $a$ and $b$ are parallel: in this case, the offset lines are dilation lines.

Finally, the problem may be solved by use of analytical geometry (method 3). This method applied to the dike shown in Fig. 2(a) is as follows. (1) Orthogonal axes are drawn arbitrarily, for example $x$ = east, $y$ = north, $z$ = vertical, as shown. (2) $x$, $y$, $z$ intercepts of all planes ($A$, $B$, $C$, $A'$, $B'$, $C'$) are measured or calculated. (3) By use of the general equation of a plane,

$$ax + by + cz = 1,$$

where $x$, $y$, $z$ intercepts are $a^{-1}$, $b^{-1}$, $c^{-1}$, respectively, the equations for all planes are determined. (4) The co-ordinates of the points of intersection of planes $A$, $B$, $C$ and of planes $A'$, $B'$, $C'$ are then obtained by calculation. For example, the point of intersection of planes

$$A\colon\ a_1x + b_1y + c_1z = d_1$$

$$B\colon\ a_2x + b_2y + c_2z = d_2$$

and

$$C\colon\ a_3x + b_3y + c_3z = d_3$$

has co-ordinates

$$x = \frac{1}{D}\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \quad y = \frac{1}{D}\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix},$$

$$z = \frac{1}{D}\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix},$$

where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}.$$

For guidance in this part of the calculation, see Trim (1983, p. A-12) and Agterberg (1974, Chap. 3). (5) The azimuth, plunge and length of the line joining the two intersection points (i.e. the measurable dilation line) can then be obtained from:

angle ($\theta$) between $y$-axis and trace of dilation line

$$\theta = \tan^{-1}[(x' - x)/(y' - y)],$$

length ($s$) of dilation line:

$$s = [(x' - x)^2 + (y' - y)^2 + (z' - z)^2]^{1/2},$$

plunge angle ($\phi$) of dilation line:

$$\phi = \sin^{-1}[(z' - z)/s].$$

With regard to dike C (Fig. 2a) the points of intersection obtained by this method are (in m):

| | $x$ | $y$ | $z$ |
|---|---|---|---|
| planes $A$, $B$, $C$ | +2.75 | +1.30 | +4.34 |
| planes $A'$, $B'$, $C'$ | +3.08 | +1.42 | +4.46 |

The calculated azimuth, plunge and length of the dilation line are 250°/20° and 0.37 m; this may be compared with 245°/28° and 0.38 m obtained by method 2 (above).

These differences are attributed to various sources of error, which have not been fully defined. It is clear, however, that errors result from (1) curvature or deflection in the planes being considered, (2) measurement error (angles and lengths) and (3) drafting error. Method 3 can involve fewer measurements and is free from drafting error, and it is therefore expected to be more accurate and precise than the others. An additional advantage of method 3 is that it is amenable to solution by computer.

The dilation line is the resultant of a dilation path which could be straight, curved or irregular. A description of the dilation path and the rate of dilation form additional structural problems.

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